

Multiparton production @ NLO with BlackHat and Sherpa



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Fermilab Theory Seminar, 10/15/2009

Based on 0907.1984 [hep-ph]
0902.2760 [hep-ph]
0709.2881 [hep-ph]

Outline

- ▶ Motivation
- ▶ Calculation of cross sections
- ▶ Techniques & implementation
 - SHERPA
 - BlackHat
- ▶ Physics Results ($W+3$ jet)
 - Tevatron
 - LHC

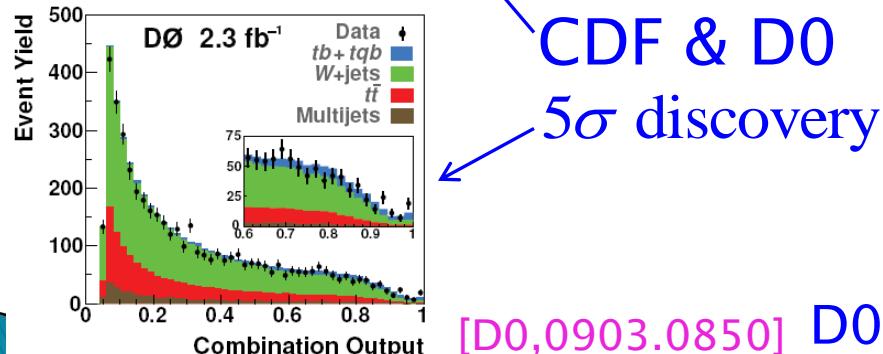
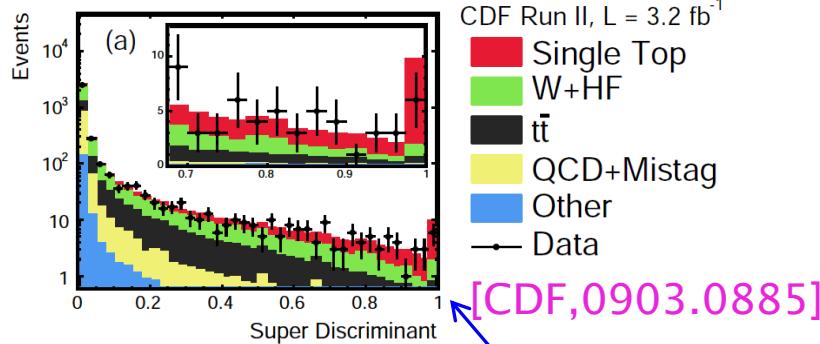
Motivation

- ▶ Many discoveries rely on precise theoretical predictions of signals and backgrounds

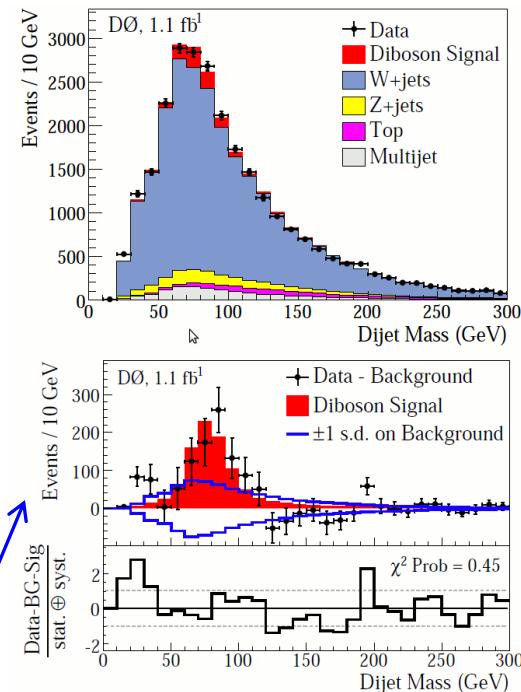
WW / WZ

[D0, 0810.3873]

Single Top



CDF & D0
5σ discovery

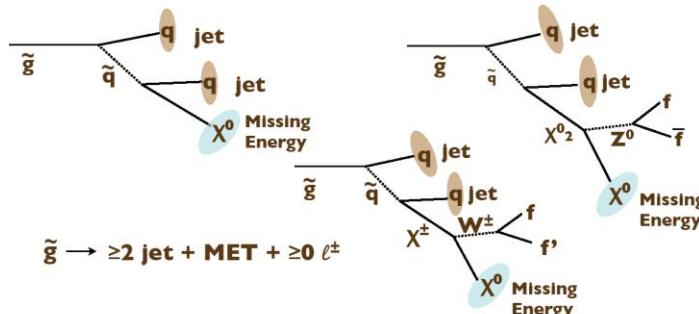


[D0, 0903.0850] D0, 4.4σ evidence

Motivation

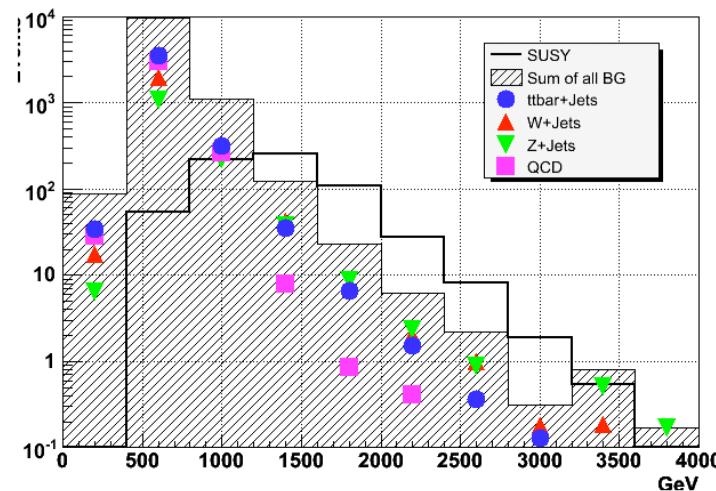
- ▶ Many discoveries rely on precise theoretical predictions of signals and backgrounds

Susy search at LHC



[M.L.Mangano, 2008]

[Atlas Collaboration]



Multijet+missing ET final state

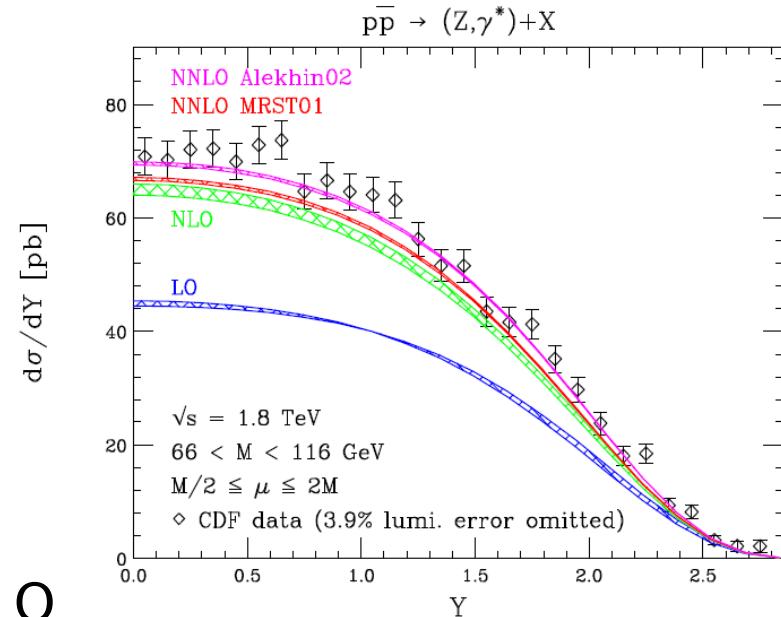
Motivation

- Tree-level (LO) predictions only qualitative estimates due to poor convergence of expansion in strong coupling
- NLO corrections can be very large (30–100% of LO)
- Matching algorithms can improve shapes, but normalization is still LO
- Scale uncertainties:
 $(W + n \text{ jets})$

n	LO	NLO
1	16%	7%
2	30%	10%
3	42%	12%

- Need NLO for reliable predictions!

[Anastasiou, Dixon,
Melnikov, Petriello, 2004]



NLO wishlist (Les Houches 2009)

Process ($V \in \{Z, W, \gamma\}$)	Comments	Motivation
pre Les Houches 2007	(completed)	
1. $pp \rightarrow VV$ jet 2. $pp \rightarrow \text{Higgs}+2\text{jets}$ 3. $pp \rightarrow VVV$ 4. $pp \rightarrow t\bar{t} b\bar{b}$ 5. $pp \rightarrow W+3\text{jets}$	$V = Z$ cases missing, W -decays included NLO QCD+EW to VBF γ cases missing $m_b = 0$, no t -decay W -decay included	Higgs background new physics background background for $t\bar{t}H$ new physics background
Les Houches 2007	(in progress)	
6. $pp \rightarrow t\bar{t}+2\text{jets}$ 7. $pp \rightarrow WW b\bar{b}$, 8. $pp \rightarrow VV+2\text{jets}$ 9. $pp \rightarrow b\bar{b}b\bar{b}$	V -decays useful	relevant for $t\bar{t}H$ relevant for $t\bar{t}$ benchmark process $VBF \rightarrow H \rightarrow VV$ Higgs and new physics signatures
Les Houches 2009		
14. $pp \rightarrow W+3\text{jets}$ 15. $pp \rightarrow Wb\bar{b}j$ 16. $pp \rightarrow jjjj$ 17. $pp \rightarrow t\bar{t}t\bar{t}$ 18. $pp \rightarrow Wjjjj$ 19. $H \rightarrow f\bar{f}f'\bar{f}'$	W -decay included $m_b = 0$ sufficient (?) leading color sufficient (?) NLO EW+QCD (completed)	new physics background Higgs search new physics background new physics background new physics background Higgs search

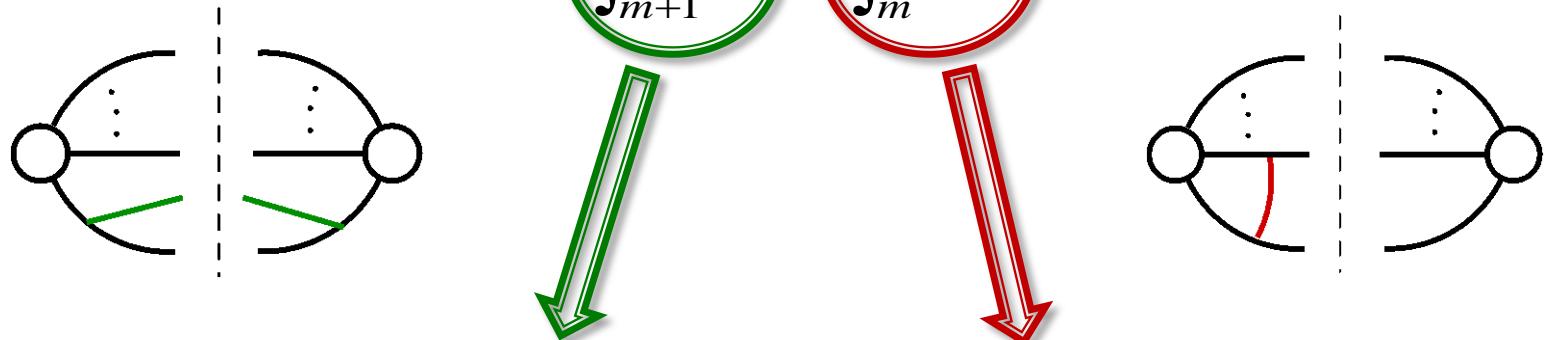
Still way to go....

Calculation of cross sections

► NLO:

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$



Real correction:

- Radiation of additional parton
- Divergent in soft and collinear limits

Virtual correction:

- Loop amplitudes
- Infrared poles after integration over loop momentum

➤ Sum is free of divergences, but cancellation only after integration over phase space!

Calculation of cross sections

Construct finite integrands: Subtraction method

- Introduce subtraction term $d\sigma^A$

$$\begin{aligned}\sigma^{NLO} &= \int_{m+1} d\sigma^R - \int_{m+1} d\sigma^A + \int_m d\sigma^V + \int_{m+1} d\sigma^A \\ &= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m (d\sigma^V + \int_1 d\sigma^A)\end{aligned}$$

A green arrow points from the $d\sigma^A$ term in the first equation to the $d\sigma^A$ term in the second equation. A red arrow points from the same $d\sigma^A$ term in the first equation to the $\int_1 d\sigma^A$ term in the second equation.

Cancels soft/collinear singularities of the real correction

Simple enough to be integrated analytically over one-parton emission in dimensional regularization:

$$\int_1 d\sigma_\varepsilon^A = \varepsilon^{-2} d\sigma^{(A,2)} + \varepsilon^{-1} d\sigma^{(A,1)} + d\sigma^{(A,0)} + O(\varepsilon)$$

Poles cancel with virtual correction

Calculation of cross sections

Construct finite integrands: Subtraction method

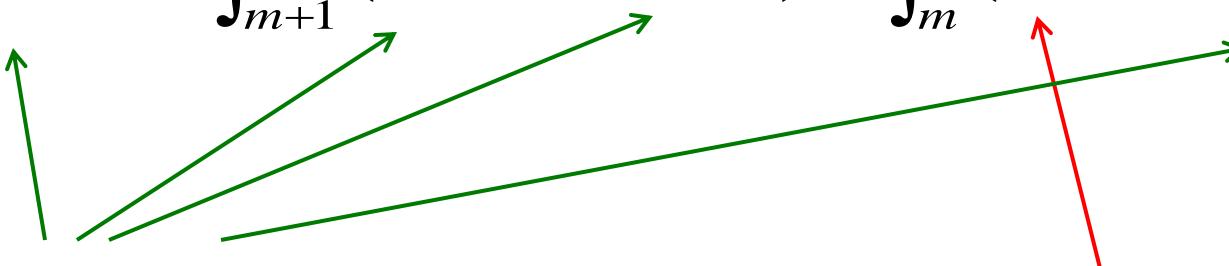
- Introduce subtraction term $d\sigma^A$

$$\begin{aligned}\sigma^{NLO} &= \int_{m+1} d\sigma^R - \int_{m+1} d\sigma^A + \int_m d\sigma^V + \int_{m+1} d\sigma^A \\ &= \underbrace{\int_{m+1} (d\sigma^R - d\sigma^A)}_{\text{Integrands are finite}} + \underbrace{\int_m (d\sigma^V + \int_1 d\sigma^A)}_{\text{suitable for numerical integration}}\end{aligned}$$

Integrands are finite and suitable for numerical integration over phase space

NLO with BlackHat & Sherpa

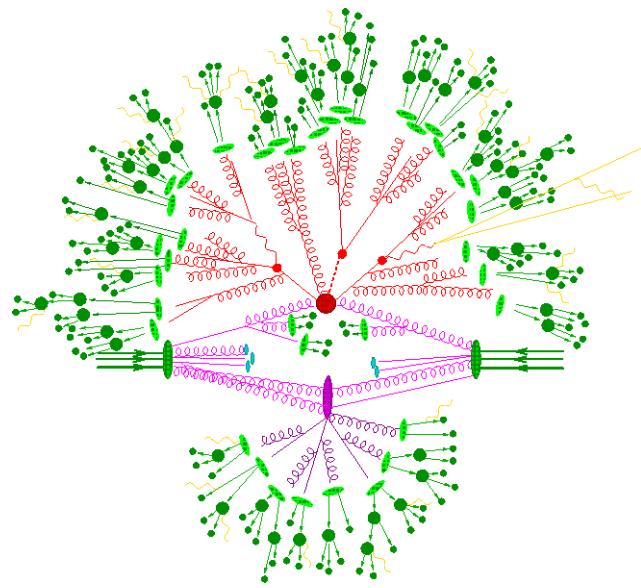
$$\sigma = \int_m d\sigma^B + \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m (d\sigma^V + \int_1 d\sigma^A)$$





SHERPA

- ▶ SHERPA is a full event generator, combining a number of perturbative and non-perturbative approaches to simulate high energy collisions
[TG, Hoeche, Krauss, Schoenherr, Schumann, Siegert, Winter]
- ▶ Here just parts of the framework are used:
 - The automated tree-level matrix element generator AMEGIC++, includes automated dipole subtraction
[TG, Krauss]
 - Phase space integration techniques
 - The event generation framework and the ANALYSIS package to evaluate generated events
- ▶ Now public: Sherpa v1.2 [www.sherpa-mc.de]

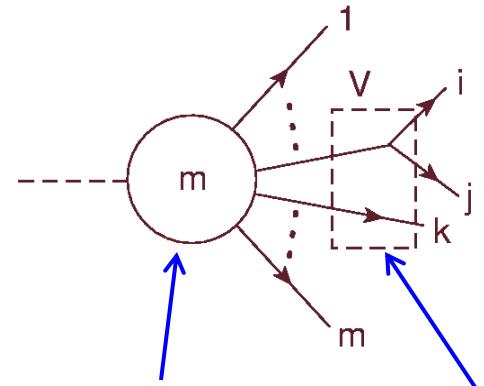


Automated dipole subtraction

[S. Catani, M.H. Seymour, 1997]

[TG, F. Krauss, 2007]

Subtraction term: $d\sigma^A = \sum_{i \neq j \neq k} d\sigma_{ij,k}^A$ (for $m+1$ -parton real correction)



Momentum map: $p_i, p_j, p_k \rightarrow \tilde{p}_{ij}, \tilde{p}_k$

$$d\sigma_{ij,k}^A = d\sigma_{\tilde{i}\tilde{j},\tilde{k}}^{LO} \otimes dV_{ij,k}$$

$$dV_{ij,k} \sim \frac{1}{p_i p_j} \mathbf{T}_{ij} \cdot \mathbf{T}_k \mathbf{V}_{ij,k} d\Phi^{(1)}$$

Integrated subtraction term: $\int_{m+1} d\sigma_{ab}^A(p_a, p_b) = \int_m \left[d\sigma_{ab}^B(p_a, p_b) \otimes \mathbf{I}(\varepsilon) \right]$

(in $4 - 2\varepsilon$ dimensions)

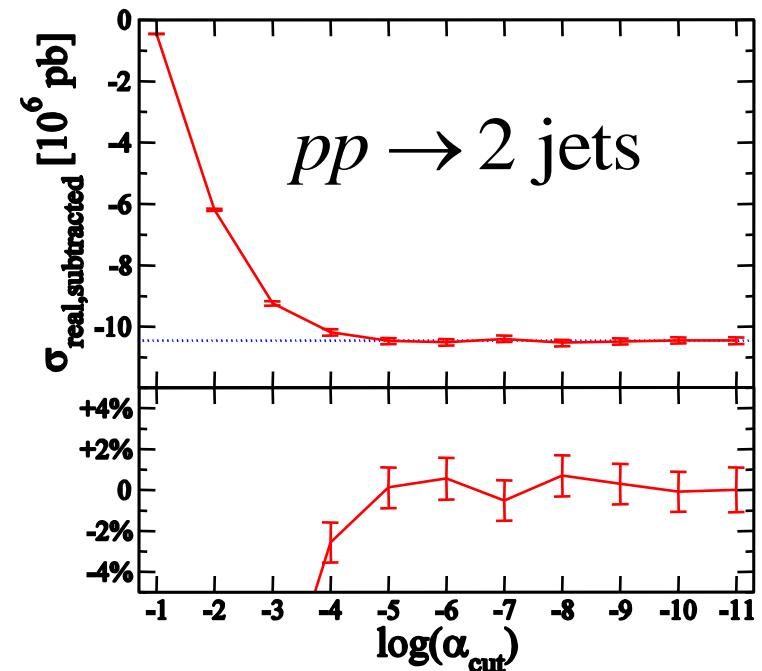
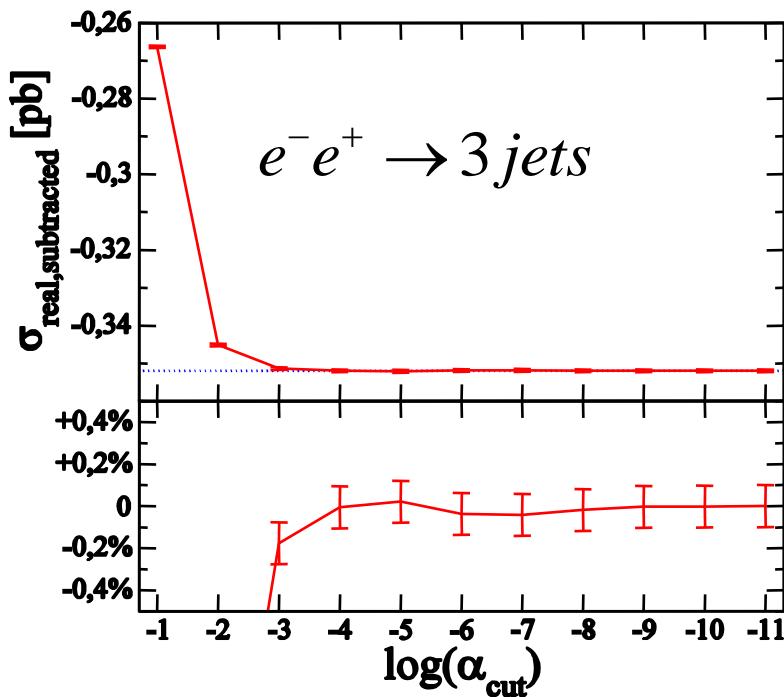
$$\mathbf{I}(\varepsilon) = -\frac{\alpha_s}{2\pi} \frac{2}{\Gamma(1-\varepsilon)} \sum_i \frac{1}{\mathbf{T}_i^2} V_i(\varepsilon) \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{4\pi\mu^2}{2p_i p_j} \right)^\varepsilon$$

Convergence and consistency checks

Cutoff dependence of
subtracted real correction:

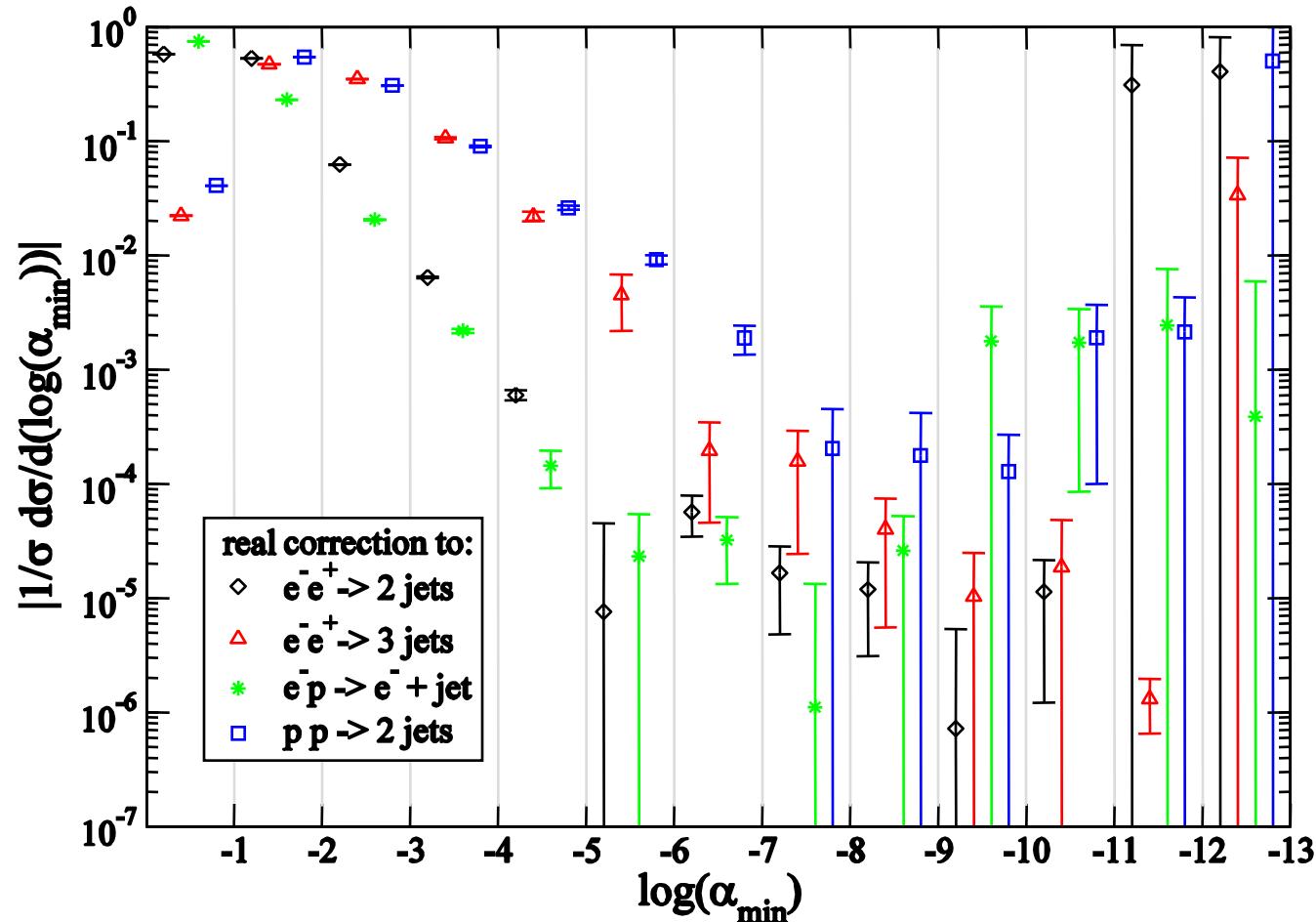
$$\alpha_{\min} = \min(\alpha_{\text{dipole}}) < \alpha_{\text{cut}}$$

i.e. FF-dipole: $\alpha_{\text{dipole}} = y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$



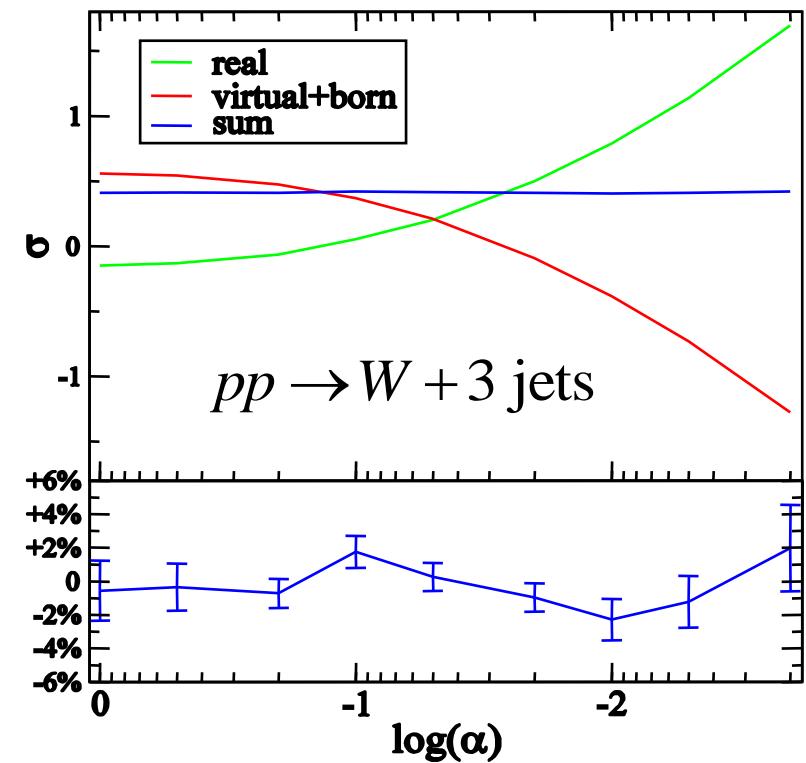
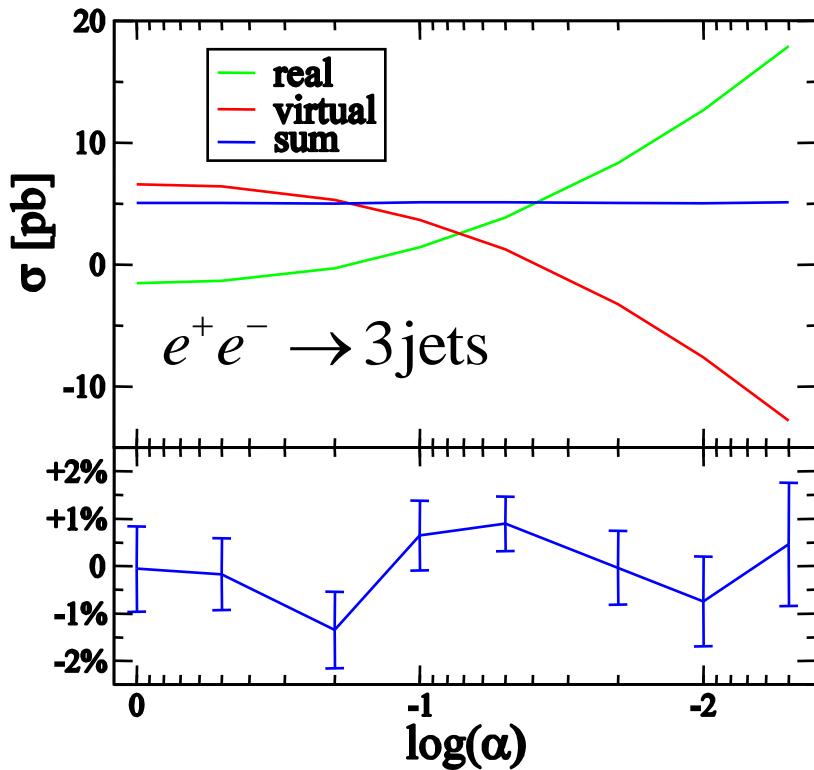
Convergence and consistency checks

Cutoff dependence of subtracted real correction:



Convergence and consistency checks

Consistency check with modified dipole terms: $d\sigma^{A'} = d\sigma^A \theta(\alpha - y)$



Comparisons with other codes

- ▶ With M. Seymour's code DISENT:

$$e^- e^+ \rightarrow 3\text{jets}, \quad e^- p \rightarrow e^- + 2\text{jets}$$

- Explicit comparison of dipole terms
- Comparison of the insertion operators I , K , P
- Checks of integrated results for real (subtracted) correction and integrated dipole terms
- ▶ Comparison of dipole terms for $pp \rightarrow ZZ + \text{jet}$ with a code by N. Kauer
- ▶ Checks of full NLO results for a number of processes with MCFM

Automatic Dipole Subtraction in SHERPA

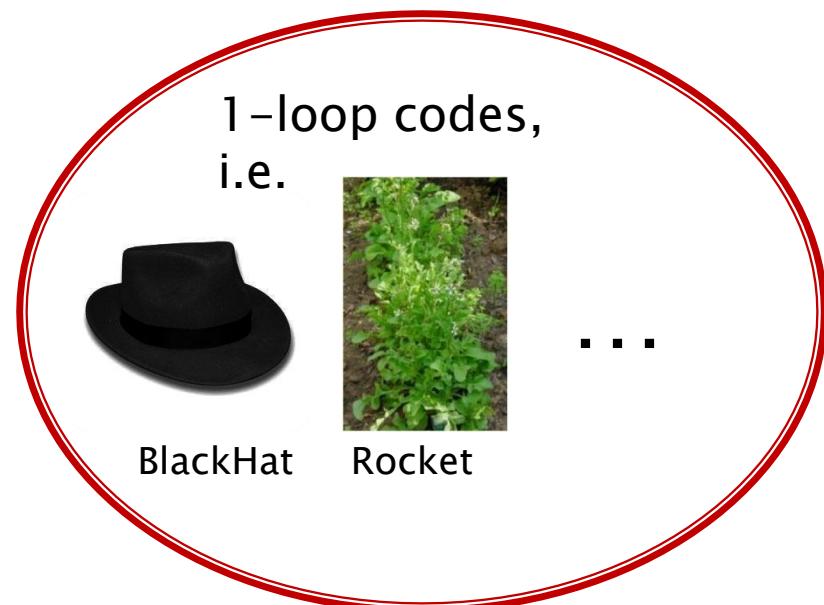
$$\sigma = \int_m d\sigma^B + \int_{m+1} \left[d\sigma^R - d\sigma^A \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]$$


Interface: 2 steps

1. Initialization

Agreement about:

- Required processes
- Model parameters
- Renormalization schemes
- Sampling parameters
- ...

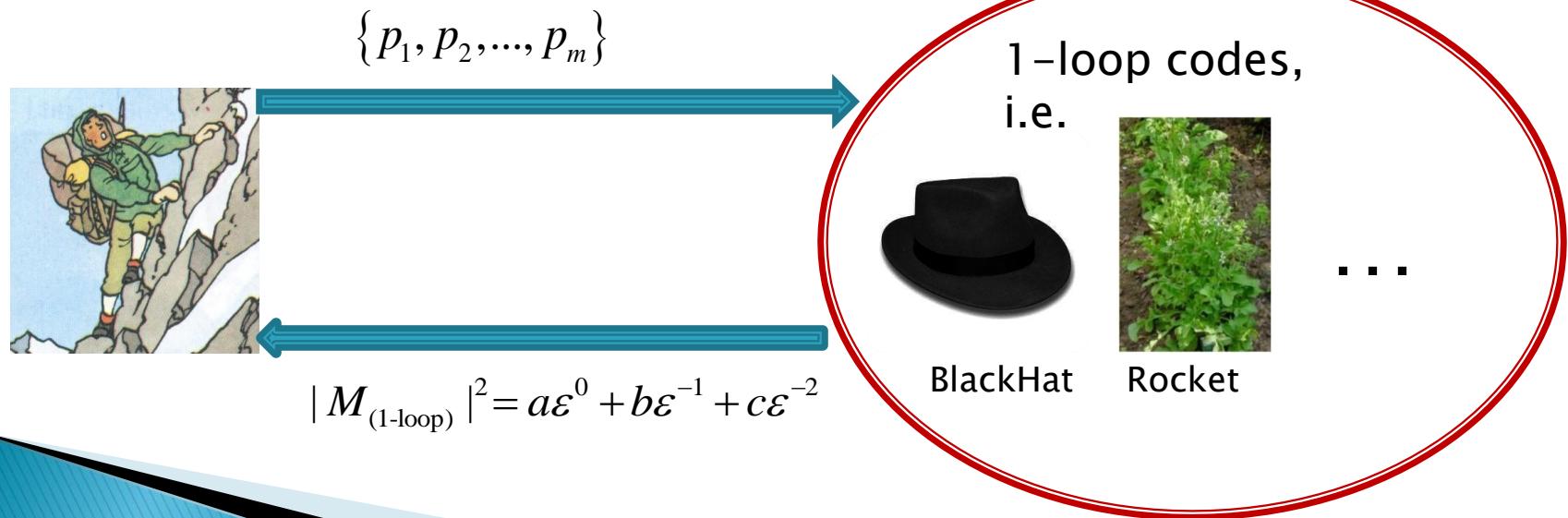


Automatic Dipole Subtraction in SHERPA

$$\sigma = \int_m d\sigma^B + \int_{m+1} \left[d\sigma^R - d\sigma^A \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]$$


Interface: 2 steps

2. Run time



Automatic Dipole Subtraction in SHERPA

$$\sigma = \int_m d\sigma^B + \int_{m+1} \left[d\sigma^R - d\sigma^A \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]$$


Interface: 2 steps

$$| M_{(1\text{-loop})}(\{p_i\}) |^2 d\phi$$

2. Run time

Interface details: see (still preliminary)
Les Houches 2009 accord

[<http://www.lpthe.jussieu.fr/LesHouches09Wiki/index.php/Draft>]

$$| M_{(1\text{-loop})} | - \alpha\varepsilon + \beta\varepsilon + \gamma\varepsilon$$

BlackHat



[Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maître]

- ▶ Goal: Automating computation of QCD one-loop amplitudes
- ▶ Use of unitarity techniques
 - [Bern, Dixon, Dunbar, Kosower, 1994]
 - [Berger, Bern, Dixon, Forde, Kosower, 2006]
 - [Ossola, Papadopolous, Pittau; Forde, 2007]
 - Work with gauge invariant on-shell objects
 - Allows recursive calculation
 - Better scaling with increasing number of external legs
(compared to traditional Feynman diagrammatic approach)
- ▶ C++ framework

Main technique: unitarity bootstrap

Decomposition of one-loop amplitudes into coefficients of scalar integrals and rational terms:

$$A = R + C$$

$$C = \sum_i b_i \text{ (square loop diagram)} + \sum_i c_i \text{ (triangle loop diagram)} + \sum_i d_i \text{ (elliptical loop diagram)}$$

- The coefficients b_i, c_i, d_i can be computed in $d=4$ dimensions using generalized unitarity
- The rational term is computed separately using on-shell recurrence relations

Integral coefficients from unitarity

► Unitarity cut: $\frac{1}{p^2 - m^2 + i0} \rightarrow 2\pi\delta(p^2 - m^2)$

$$\text{Diagram} = R + \sum_i b_i \text{Diagram} + \sum_i c_i \text{Diagram} + \sum_i d_i \text{Diagram}$$

► Apply cuts on both side of the equation

$$\text{Diagram} = b \text{Diagram}$$

$$\text{Diagram} = c \text{Diagram} + \sum_i b_i \text{Diagram}$$

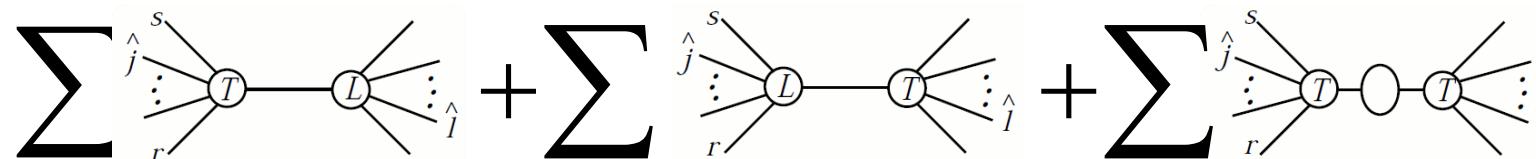
$$\text{Diagram} = d \text{Diagram} + \sum_i c_i \text{Diagram} + \sum_i b_i \text{Diagram} + \sum_i b_i \text{Diagram}$$

On-shell recursions for rational terms

- ▶ Construct rational part recursively from on-shell tree-amplitudes (T) and rational pieces of one-loop amplitudes (L)

$$R = \sum \text{Diagram } 1 + \sum \text{Diagram } 2 + \sum \text{Diagram } 3 + \text{spurious poles}$$

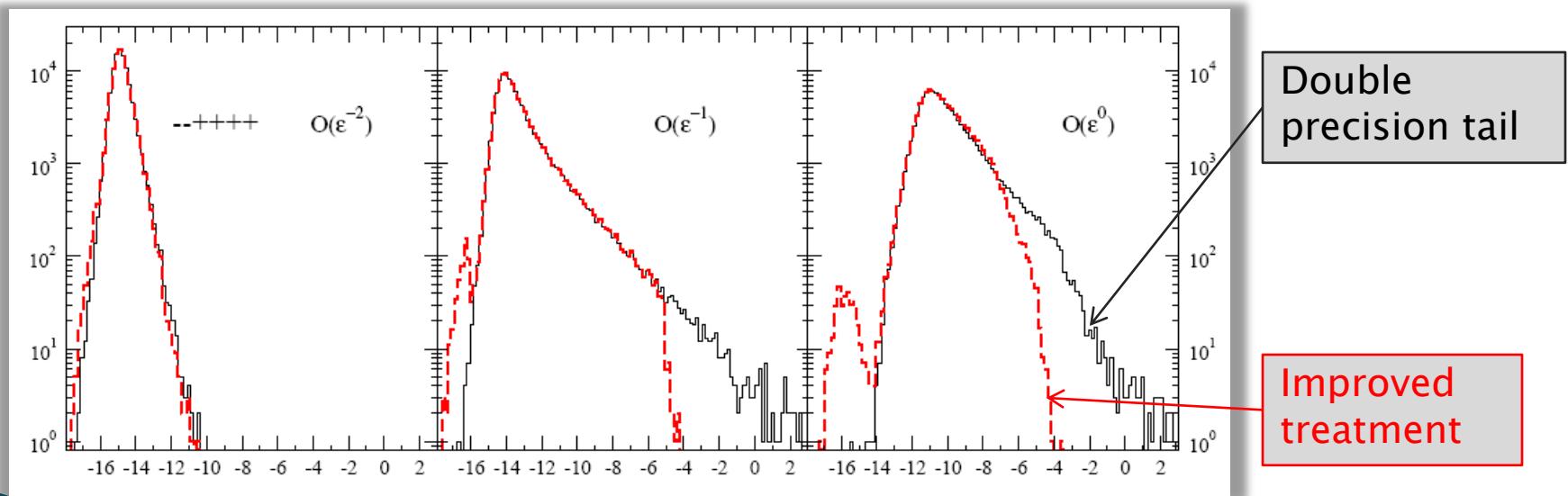
The equation shows the recursive construction of the rational part R of a scattering amplitude. It is the sum of three contributions: Diagram 1 (left), Diagram 2 (middle), and Diagram 3 (right). Each diagram consists of a tree-level vertex (a circle with a blue border) labeled T , a loop vertex (a circle with a red border) labeled L , and a one-loop vertex (an oval) labeled T . External lines are labeled with momenta s , j , r , and l . The first two diagrams have a single external line between the tree and loop vertices. The third diagram has a double line between them. The text "+spurious poles" is written in blue.



- ▶ “spurious poles” to cancel with unphysical poles in the cut part
 - Extracted numerically from the cut part (somewhat tricky and possible source for numerical inaccuracies, but works...)

Amplitudes and numerical stability

- ▶ Quantify numerical precision: $x = \log_{10} \left(\frac{|A^{\text{num}} - A^{\text{target}}|}{A^{\text{target}}} \right)$
- ▶ Improvement:
 - Perform tests to identify problematic phase space points
 - Recalculate those point with higher precision
→ Increases average computation time only by few %

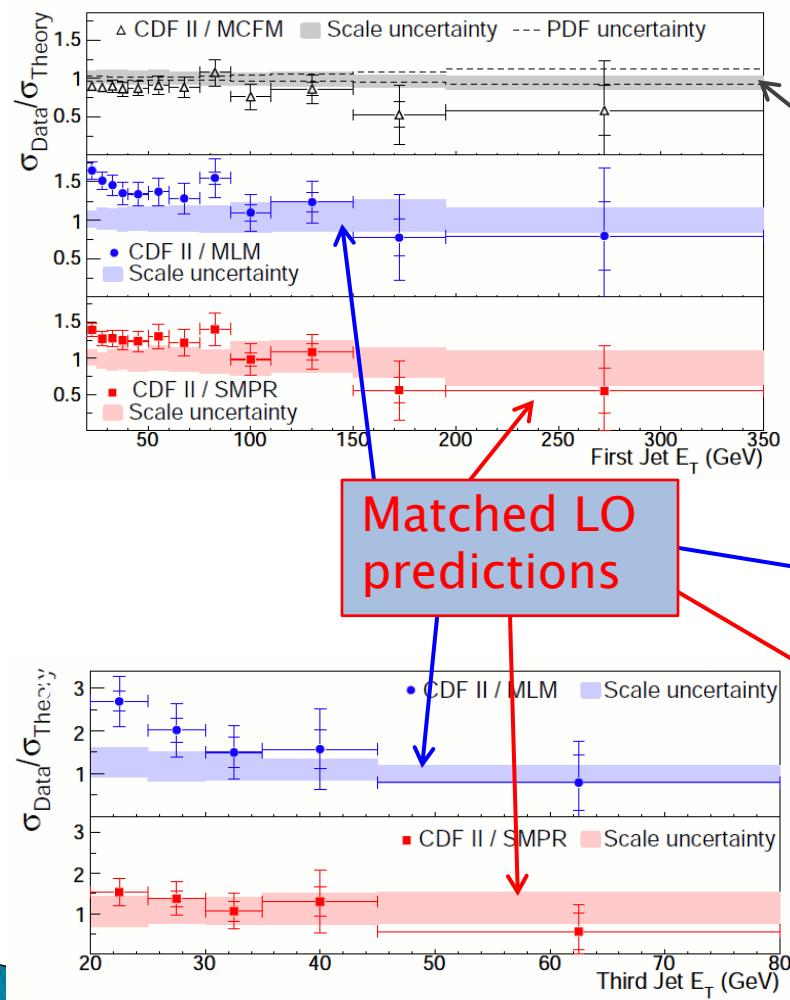


W + jets physics...

$W + \text{jets}$

- ▶ Important background process to
 - SM physics: Higgs, single top, diboson, ...
 - New physics
- ▶ NLO at cross section level
 - MCFM: $W+0/1/2$ jets [Campbell, Ellis, 2002]
 - $W+3$ jet, Leading color [Berger, Bern, Dixon, Cordero, Forde, TG, Ita, Kosower, Maître, 2009]
[Ellis, Melnikov, Zanderighi, 2009]
 - Full color [arXiv:0907.1984]

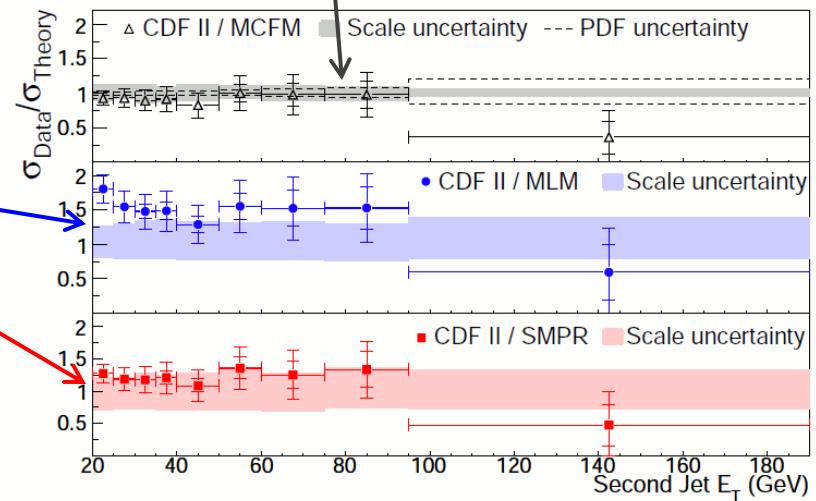
$W + n$ jets @ Tevatron



[CDF: PRD 77 011108, arXiv:0711.4044]

integrated luminosity: 320 pb^{-1}

NLO predictions
with MCFM



W + n jet @ Tevatron

Total Rates: [CDF: PRD 77 011108, arXiv:0711.4044]

# of jets	CDF	LO	LC NLO	NLO
1	53.5 ± 5.6	$41.40^{+7.59}_{-5.94}$	$58.3^{+4.6}_{-4.6}$	$57.83^{+4.36}_{-4.00}$
2	6.8 ± 1.1	$6.16^{+2.41}_{-1.58}$	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.796^{+0.488}_{-0.276}$	$0.908^{+0.044}_{-0.142}$	$0.882^{+0.057}_{-0.138}$

Results from [Ellis, Melnikov, Zanderighi, 0906.1445]: (W+3 jets)
LC: $1.01^{+0.05}_{-0.17}$ LC* R : $0.91^{+0.05}_{-0.15}$

$$E_T^{jet} > 25 GeV$$

$$|\eta^{jet}| < 2$$

$$E_T^e > 20 GeV$$

$$|\eta^e| < 1.1$$

$$E_T^{miss} > 30 GeV$$

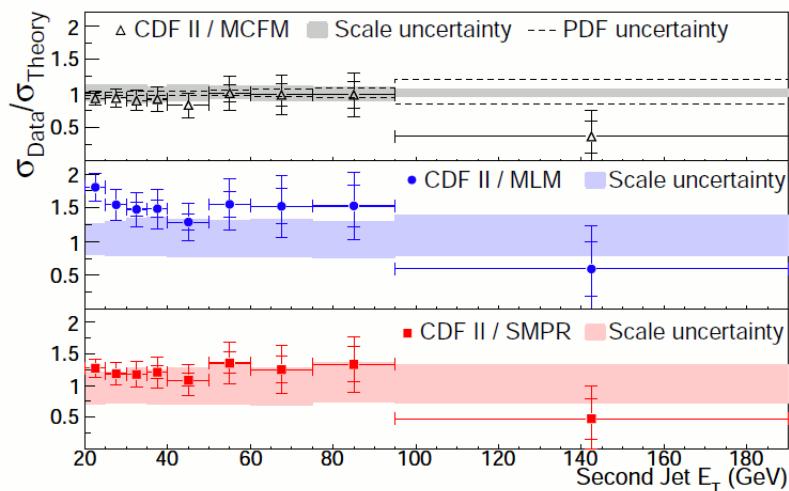
$$M_T^W > 20 GeV$$

SISCone
[G. Salam, G. Soyez]

Leading color approximation

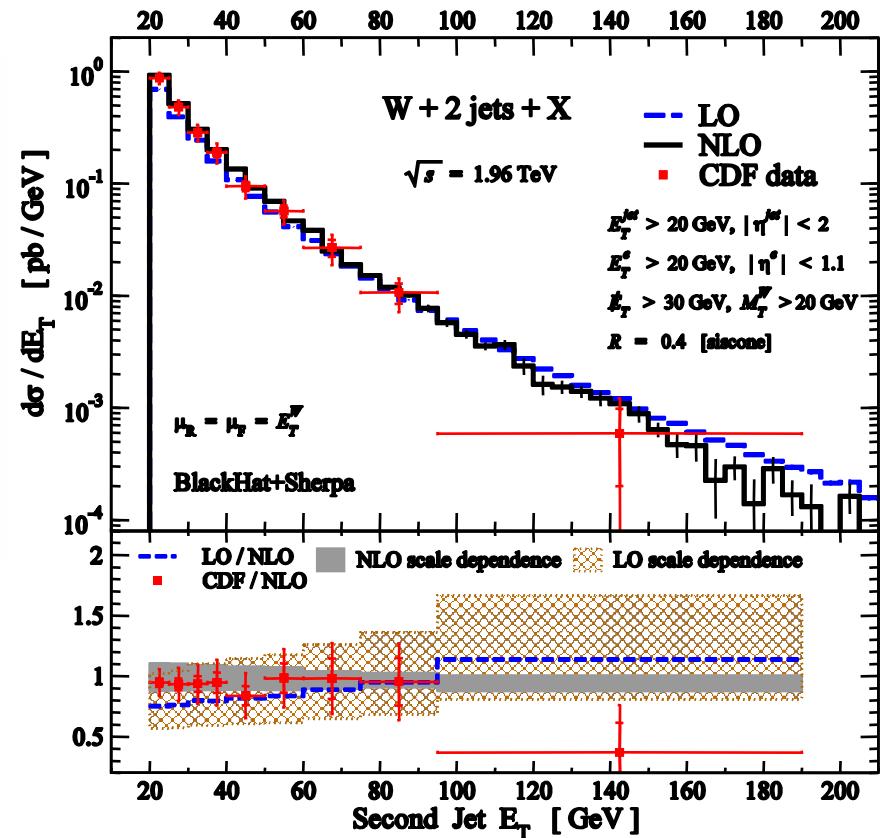
- ▶ “LC NLO” neglects subleading-color terms $\sim 1/N_C^2$ and contributions from closed fermion loops $\sim N_f/N_C$ in the **finite part of one-loop amplitudes** (which is reweighted by LO/LC LO)
- ▶ All other pieces (**born, real, subtraction**) are computed in full color
- ▶ Difference to full color in W+1/2/3 jets is just about 1–3%, which is much more expensive to compute
- ▶ Our approach: use MC sampling techniques and compute the “difference” at a much lower rate

Warmup: W + 2 jets @ Tevatron

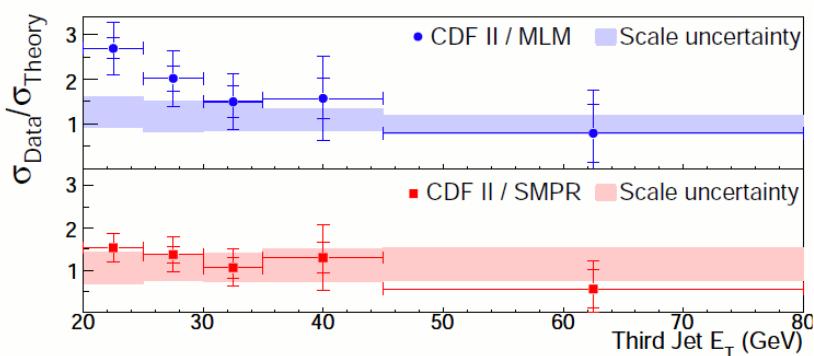


$$\mu_r = \mu_f = \sqrt{M_W^2 + p_{T,W}^2}$$

$$\mu/2 \dots 2\mu$$

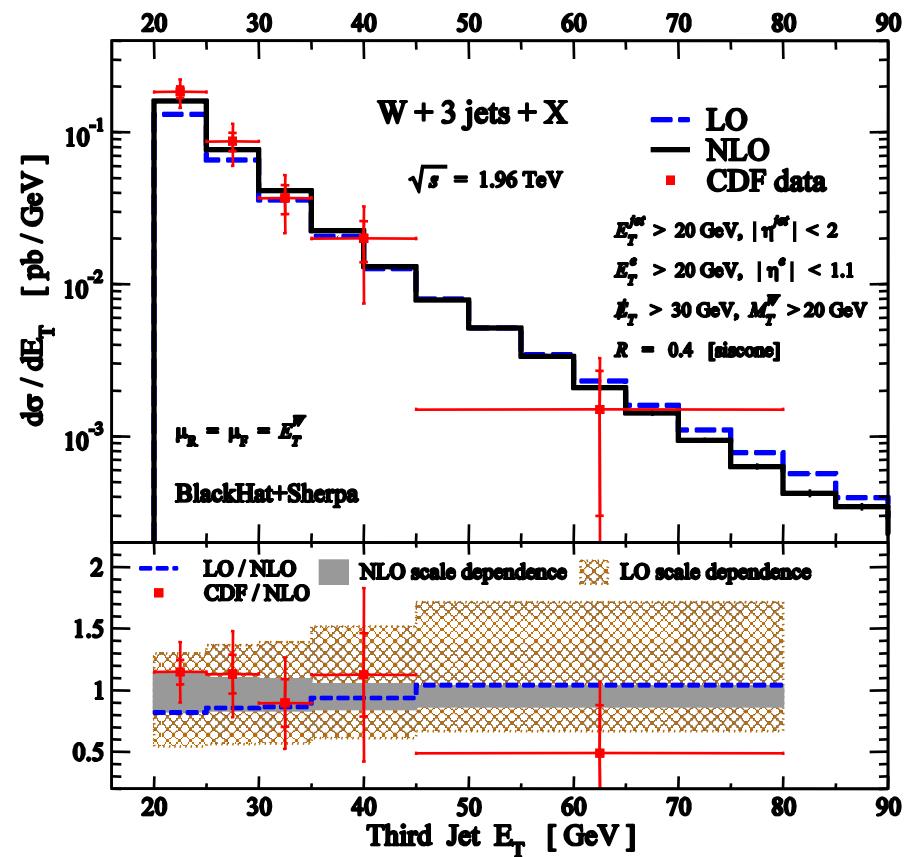


W + 3 jets @ Tevatron

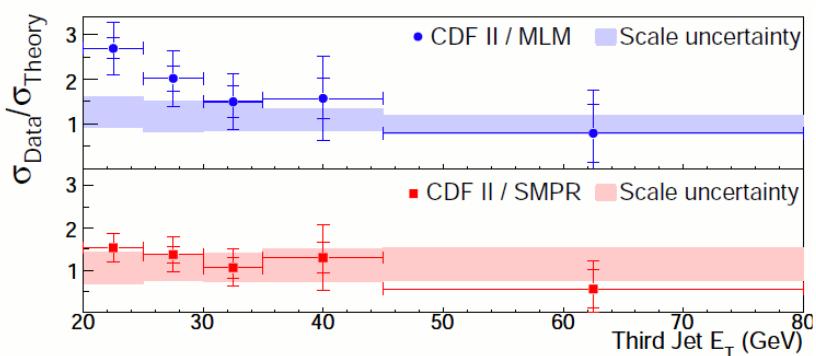


$$\mu_r = \mu_f = \sqrt{M_W^2 + p_{T,W}^2}$$

$$\mu/2 \dots 2\mu$$

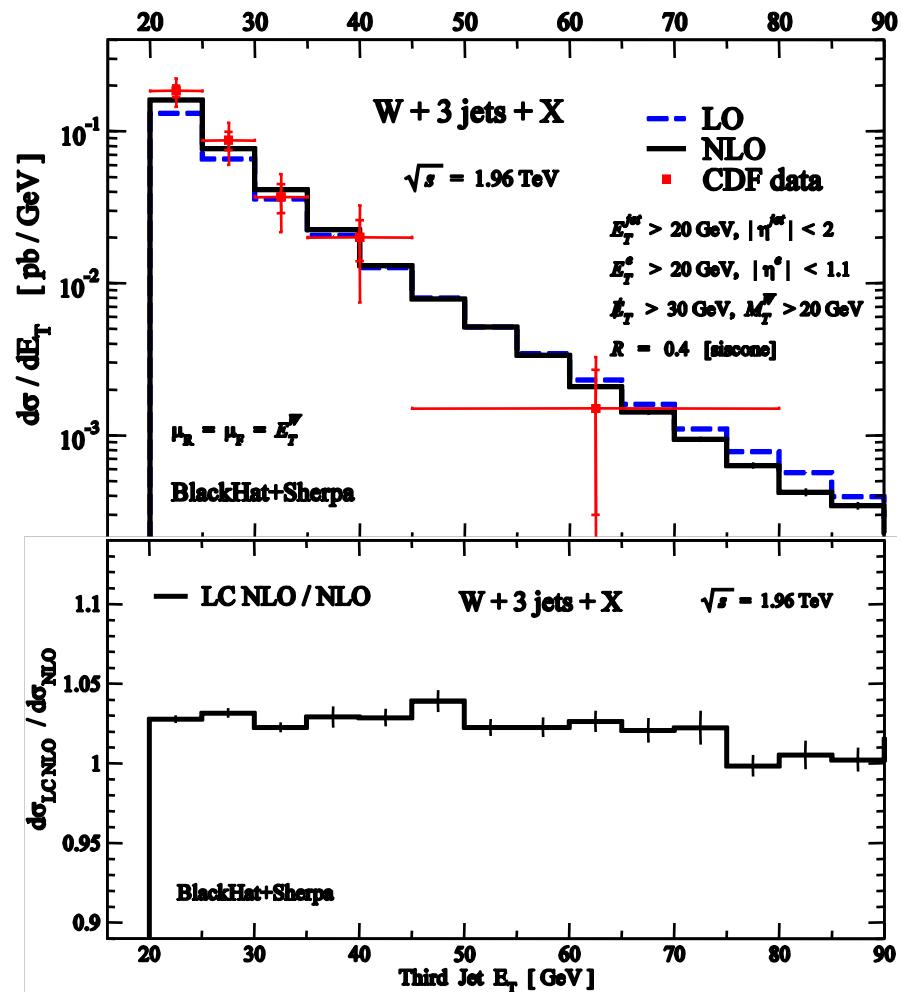


W + 3 jets @ Tevatron

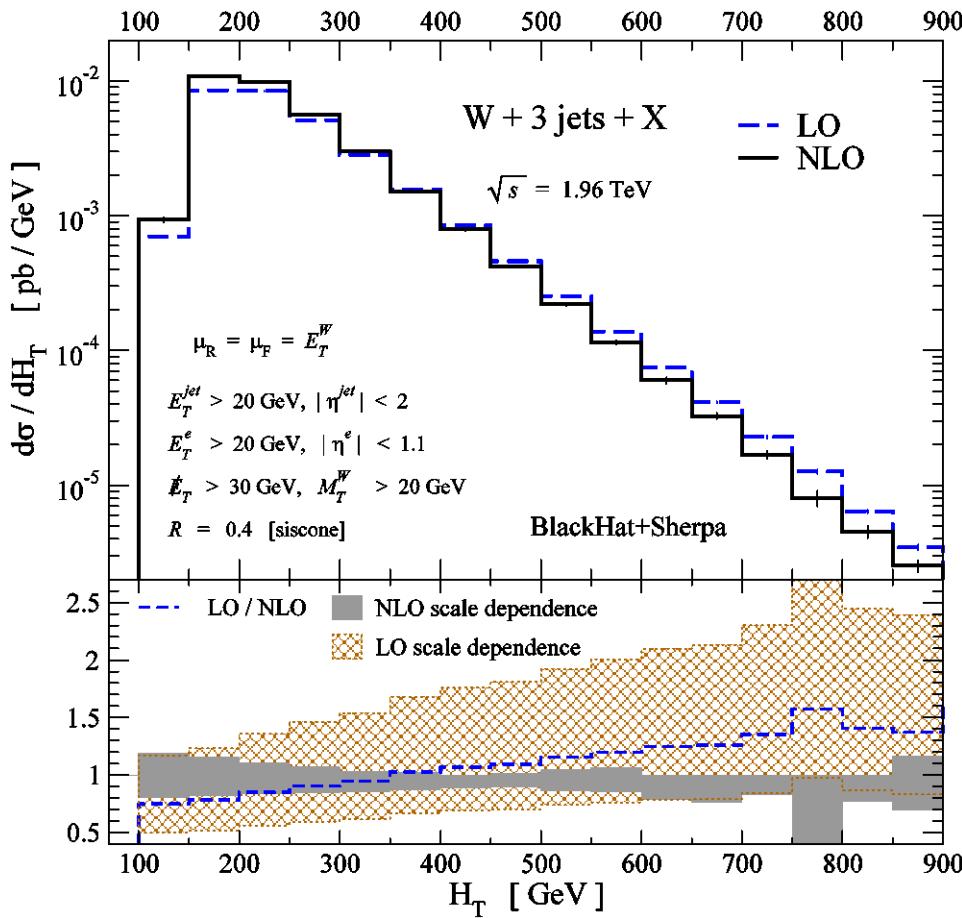


$$\mu_r = \mu_f = \sqrt{M_W^2 + p_{T,W}^2}$$

$$\mu/2 \dots 2\mu$$

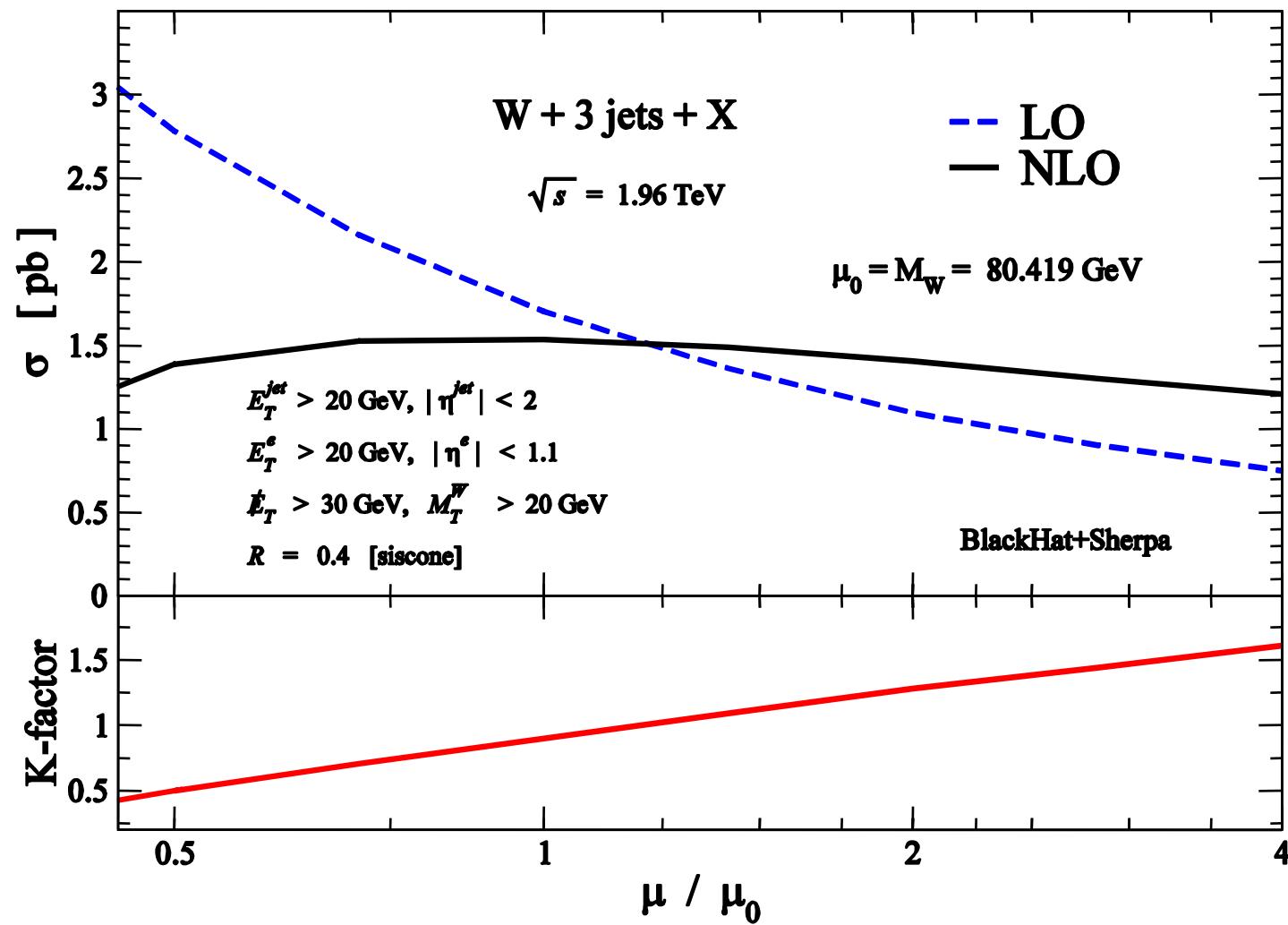


W + 3 jets @ Tevatron



$$H_T = \sum_j E_{T,j}^{jet} + E_T^e + E_T^{\text{miss}}$$

$W + 3$ jets @ Tevatron: total cross section



Predictions for the LHC

Collision energy: 14 TeV

$$E_T^{jet} > 30 GeV$$

$$|\eta^{jet}| < 3$$

$$E_T^e > 20 GeV$$

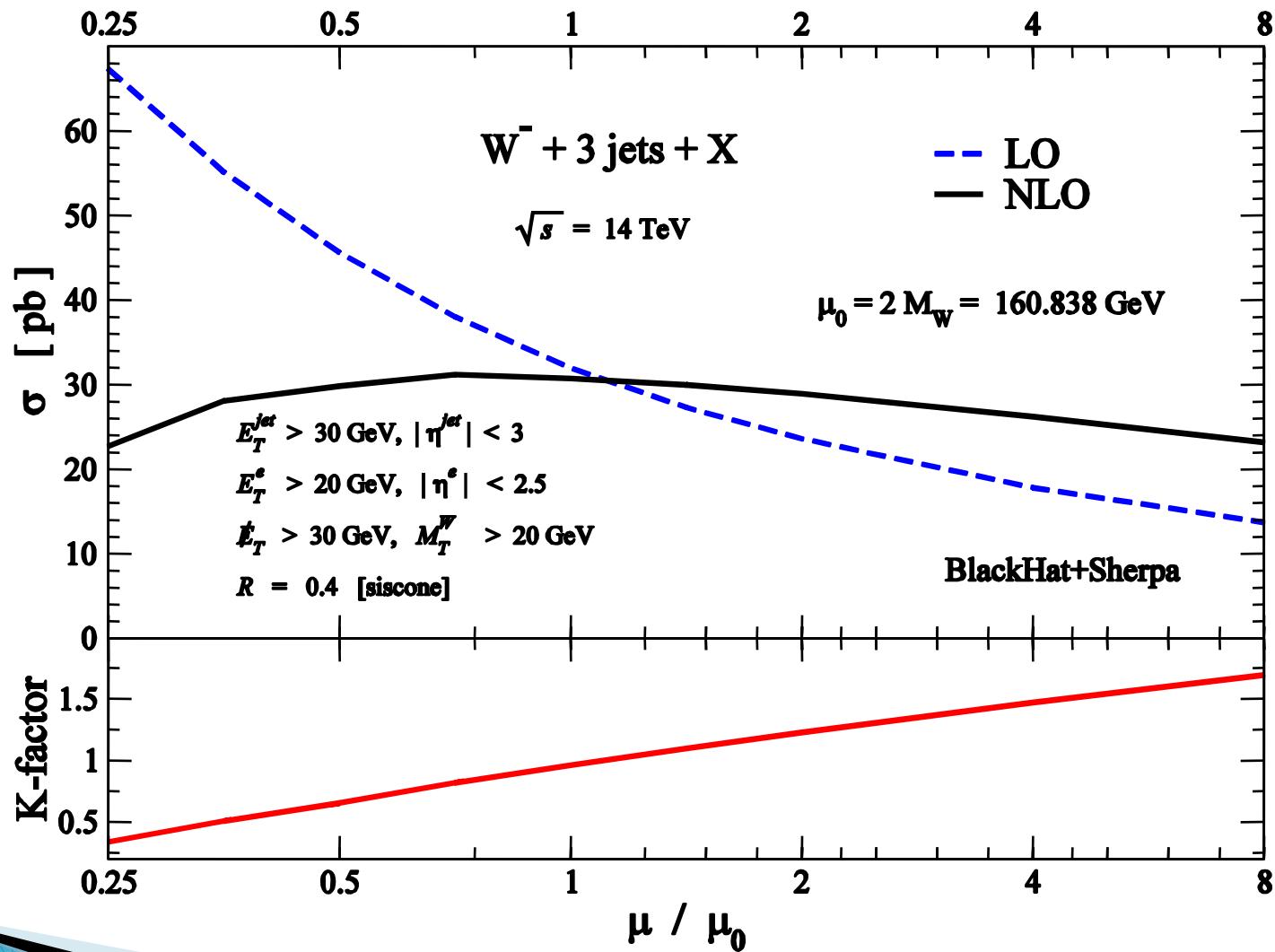
$$|\eta^e| < 2.5$$

$$E_T^{miss} > 30 GeV$$

$$M_T^W > 20 GeV$$

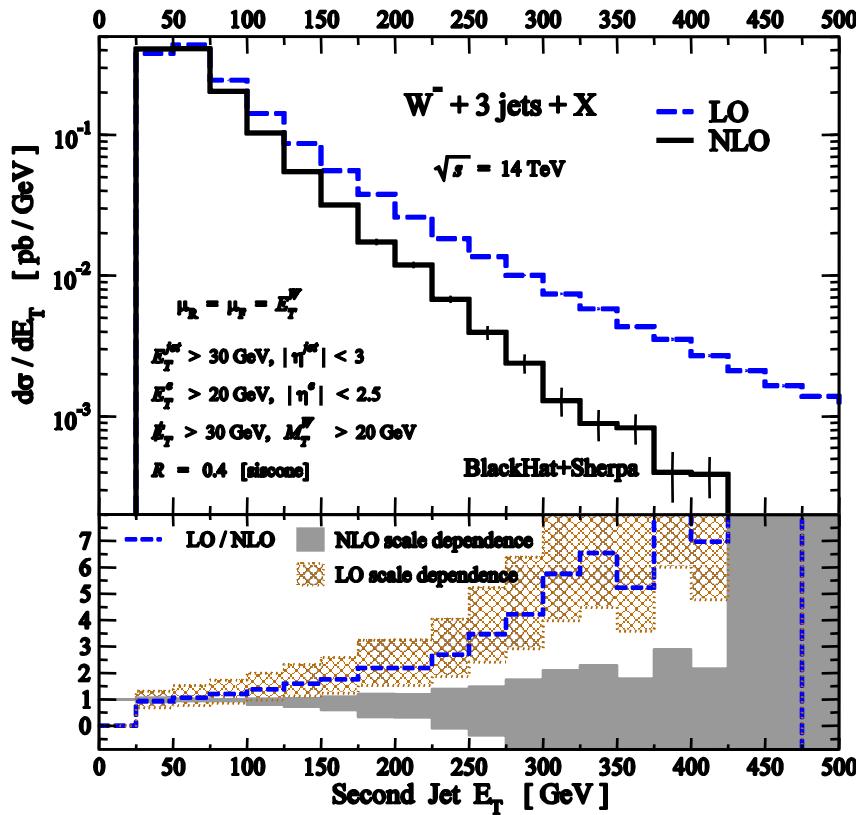
Jet algorithm: SISCone
[G. Salam, G. Soyez]

LHC results: Total cross section

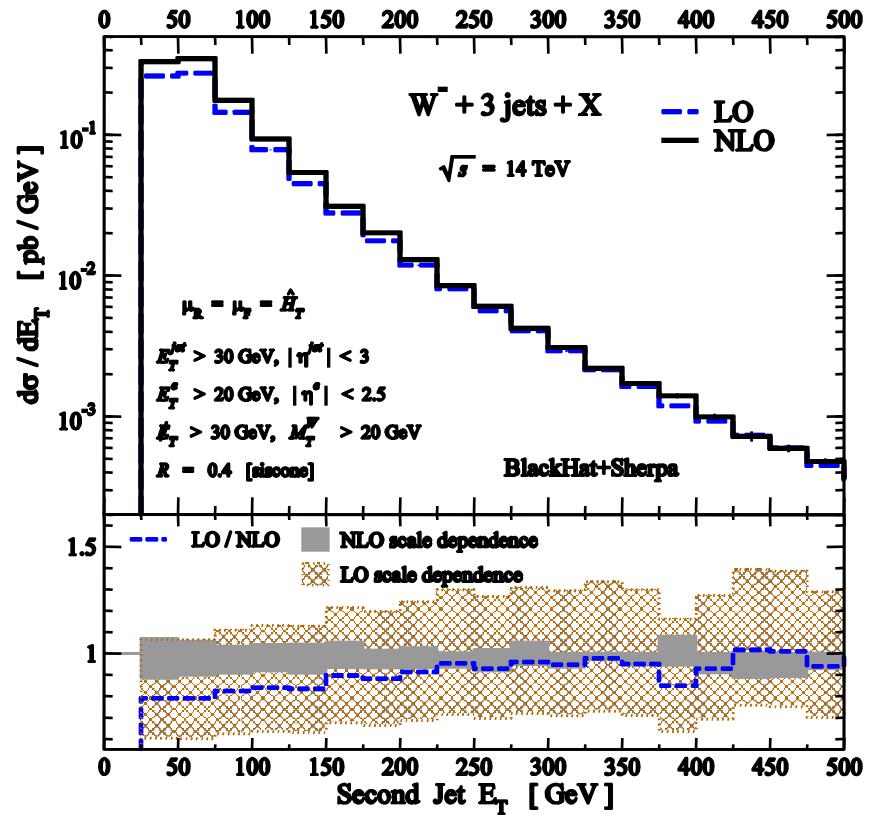


LHC results

► Scale choices



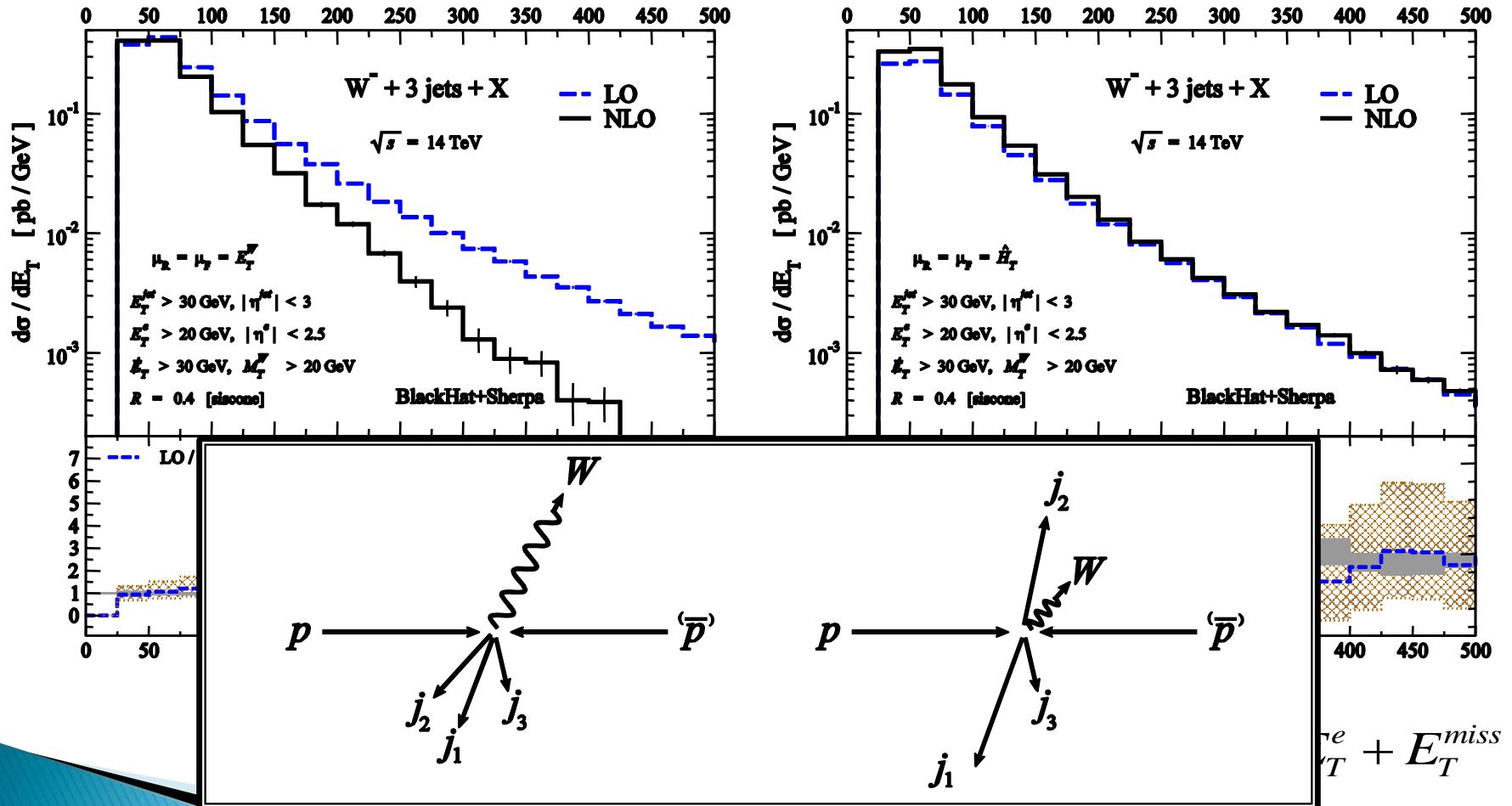
$$\mu = E_T^W = \sqrt{M_W^2 + p_{T,W}^2}$$



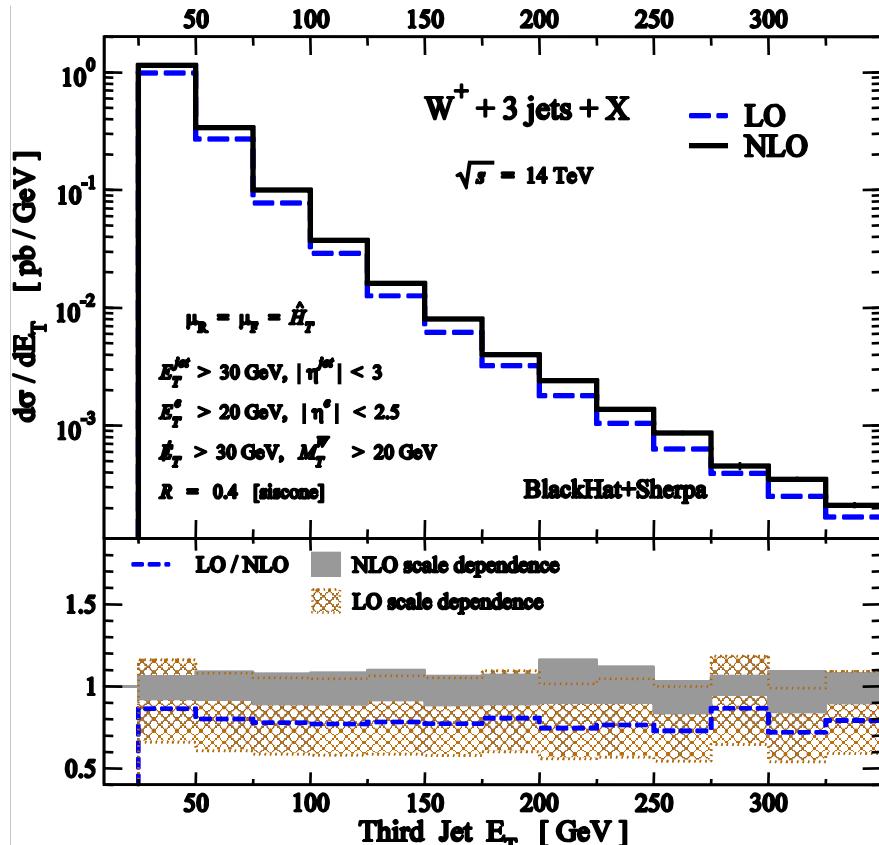
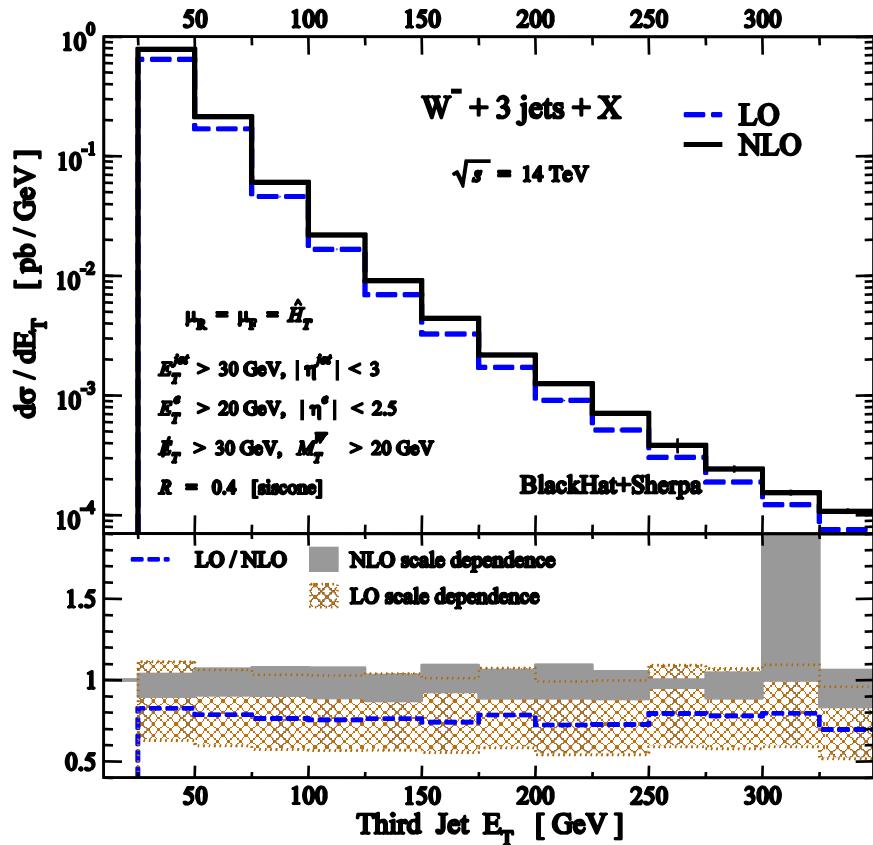
$$\mu = H_T = \sum_j E_{T,j}^{jet} + E_T^e + E_T^{\gamma}$$

LHC results

► Scale choices



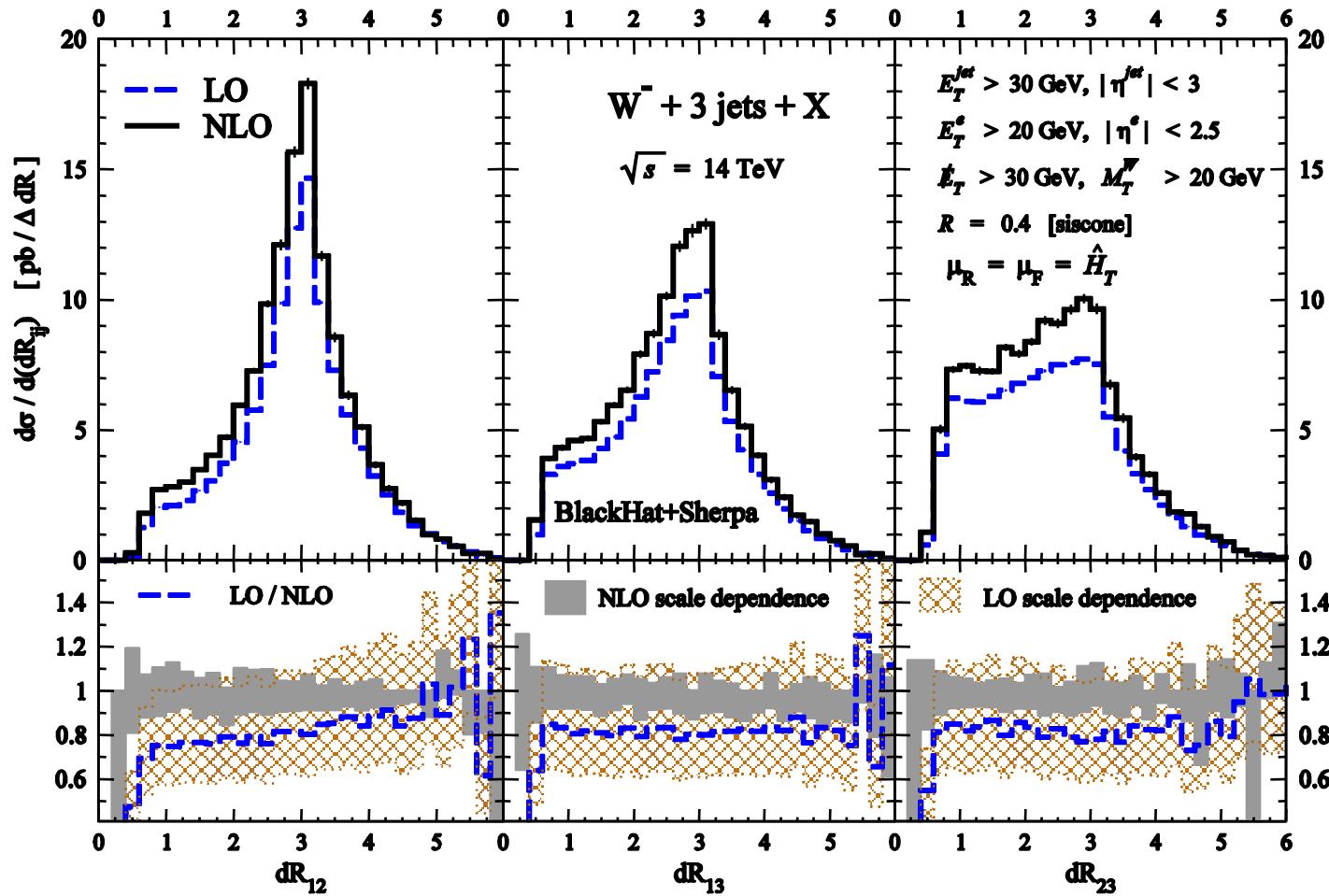
LHC results



$$\mu_r = \mu_f = H_T$$

Scale variation: $\mu/2 \dots 2\mu$

LHC results



Summary and outlook

- ▶ NLO calculations with BlackHat & Sherpa
 - New efficient computational approach to one-loop QCD amplitudes
- ▶ First results for W+3 jet to compare to data
- ▶ LHC predictions
- ▶ Future:
 - More processes (Z+3 jets, ...)
 - Improvements in speed
 - Automation

- ▶ Now released

SHERPA v1.2

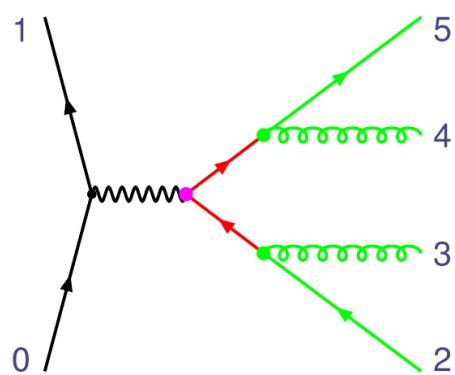
- ▶ Includes:
 - Dipole subtraction
 - Analysis framework for NLO events
 - A revised implementation of (LO) CKKW merging to work with multiple ME generators and Parton Showers
[Hoeche, Krauss, Schumann, Siegert, 2009]
 - The ME generator **COMIX** [TG, Hoeche, 2008]
 - A new Parton shower, based on CS dipoles
[Schumann, Krauss, 2009]
 - And more...



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Note to phase space integration

- For real (subtracted) correction: $\int_{m+1} (d\sigma^R - d\sigma^A)$ the $(m+1)$ -parton phase space is generated directly multi-channeling over parameterizations obtained from the (Feynman diagrammatic) structure of the real ME



- Generate a channel for each diagram using a few building blocks
 - $\sim P_0(23)P_0(45)D(23,45)D(2,3)D(4,5)$
- Adapt to the full structure of the integrand by relative weights of single channels and a VEGAS grid for each map
- For $P_0 \sim s^{-\nu}$ some caution necessary when choosing exponent ν to not introduce (integrable) singularities